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A4ria

NUMBERS

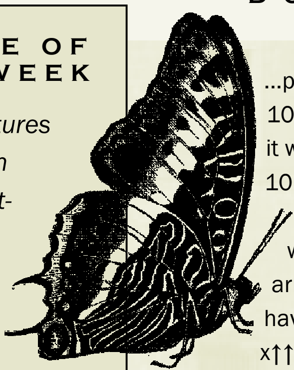
QUOTE OF THE WEEK

The structures with which mathematics deals are more like lace, the leaves of trees and the play of the light and shadow on a human face than they are like buildings and machines, the least of their representatives.
— Scott Buchanan

THE MOST REMARKABLE FORMULA IN MATHEMATICS

$$e^{i\pi}+1=0$$

Donald Knuth on Fairly Large Numbers



...people usually write 10^{10} but I like to write it with an arrow $10\uparrow 10$ because of the next step up, which uses *two* arrows. I mean, if you have two arrows as in $x\uparrow\uparrow n$, you are computing x to the x to the x to the x and so on, n times: $x\uparrow\uparrow n = x\uparrow(x\uparrow(\ldots\uparrow x)\ldots)$.

Now this arrow notation is my big claim to fame, because it's what got me into the *Guiness Book of World Records*. In particular, $10\uparrow\uparrow 10$—that's a fairly large number. If you put monkey down at a typewriter and wait until he types out the entire text of Hamlet, with no errors, the number of trials is only 1 followed by about 40,000 zeros. Our number $10\uparrow\uparrow 10$ is 1 followed by quite a few more zeros than that. The general rule, of course, is that if you have k arrows, you just define it as the operation with k-1 arrows, over and over again. I want to give you a small example of the arrow functions so that you can better understand these finite numbers. Let's look at ten quadruple-arrow three. By definition that means ten triple-arrow ten triple-arrow ten: $10\uparrow\uparrow\uparrow\uparrow 3 = 10\uparrow\uparrow\uparrow(10\uparrow\uparrow\uparrow 10)$; so we first have to evaluate ten triple-arrow ten. Well, of course, ten triple-arrow ten is

$10\uparrow\uparrow(10\uparrow\uparrow(10\ldots(10\uparrow\uparrow 10))\ldots)$

and that is

$10\uparrow\uparrow(10\ldots(10\uparrow\uparrow 10^{10}$↗stack of tens$)\ldots)$ where the stack of tens is $10\uparrow\uparrow 10$ levels tall.

Donald Knuth

It's such a huge number, I can't even write it down, but then we repeat the double-arrow operation again, getting an even huger number; and so on, until finally we get this thing evaluated.

Of course we're not done yet. This is just ten triple-arrow ten. In order to get the final number that I started with—ten quadruple-arrow three—I have to take ten triple-arrow to this number....

If you stop to think about this, you'll have to admit that, if anything is mind boggling, this is; it's *incredibly* huge. Now that we have evaluated $10\uparrow\uparrow\uparrow\uparrow 3$, let's call it Super K. If you don't agree that Super K is so large as to be beyond human comprehension, I can at least prove conclusively that if you consider all the number less than Super K, almost all of them are impossible to describe in any way in the universe. On the other hand, Super K is very small as finite number go; almost all finite number are a lot bigger than this. As soon as you begin to understand the immensity of Super K, you will realize that just being finite isn't much of a limitation, and you will see how pointless are the philosophers' discussions about finite versus infinite. Infinity is a red herring.

Strip of the week