

Aria

GÖDEL'S INCOMPLETENESS THEOREM

...Gödel's conclusion sprang not from a weakness in *Principia Mathematica* but from a strength. That strength is the fact that numbers are so flexible or "chameleonic" that their patterns can mimic patterns of reasoning. Gödel exploited the simple but marvelous fact that the familiar whole numbers can dance in just the same way as the unfamiliar symbol-patterns of *PM* dance. More specifically, the *prim* numbers that he

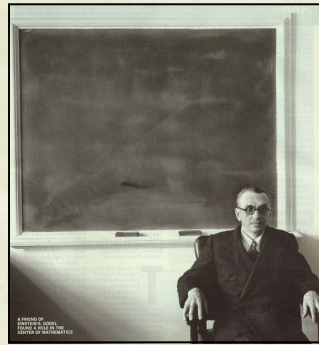
invented act indistinguishably from provable strings, and one of *PM*'s natural strengths is that it is able to talk about *prim* numbers. For this reason, it is able to talk about itself (in code). In a word, *PM*'s expressive power is what gives rise to its incompleteness. What a fantastic irony!

— Douglas Hofstadter (who else!)

NAGEL AND NEWMAN ON GÖDEL'S THEOREM

...a climate of opinion was generated in which it was tacitly assumed that each sector of mathematical thought can be supplied with a set of axioms sufficient for developing systematically the endless

very large class of deductive systems—number theory, for example—unless one adopts principles of reasoning so complex that their internal consistency is as open to doubt as that of the systems themselves. In the light of these conclusions, no final systematization of many important areas of mathematics is attainable, and no absolutely impeccable guarantee can be given that many significant branches of mathematical thought are entirely free from internal contradiction.



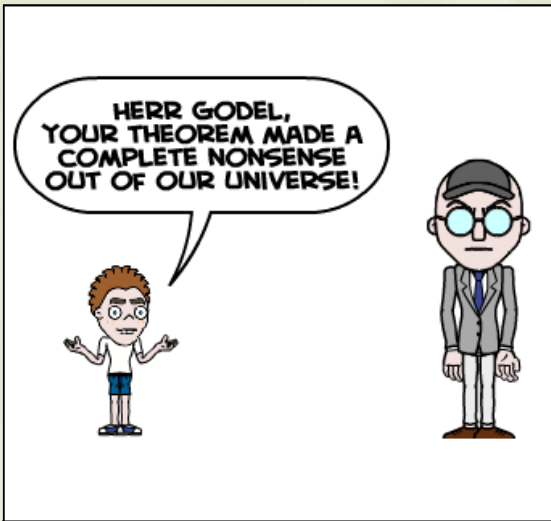
Kurt Gödel

Gödel's findings thus undermined deeply rooted preconceptions and demolished ancient hopes that were being freshly nourished by research on the foundations of mathematics.

totality of true propositions about the given area of inquiry.

Gödel's paper showed that this assumption is untenable. He presented mathematicians with the astounding and melancholy conclusion that the axiomatic method has certain inherent limitations, which rule out the possibility that even the properties of the non-negative integers can ever be fully axiomatized. What is more, he proved that it is impossible to establish the internal logical consistency of a

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One needs a meta-system to make the original one consistent and complete, but then...