A4ria

THE DIRAC EQUATION

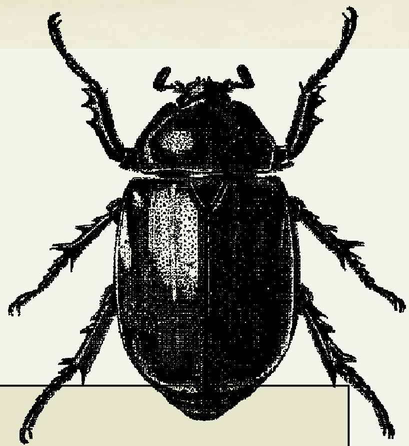
Sander Bais on the Dirac Equation

In spite of its tremendous successes, the Schrodinger equation had a serious drawback: it was not compatible with special relativity. This may be inferred from the fact that in the equation the space and time variables x and t do not appear on equal footing: it contains a first derivative with respect to time, but a second derivative with respect to the spatial coordinates. Dirac solved this problem with the equation carrying his name.

$$\left\{\gamma^{\mu}\left(i\frac{\partial}{\partial x^{\mu}}-eA_{\mu}\right)-m_{e}\right\}\Psi(x^{\nu})=0$$

The Dirac equation has quite an involved mathematical structure, which is somewhat hidden by the compact notation, so let us take some time to comment on the notation used. There is an index μ which can take the values 0,1, 2 or 3, indicating time and the three space components, indeed appearing on equal footing. The four Aμ fields, called 'electromagnetic potentials', describe the electromagnetic field in which (for example) the electron moves, and m_e is the electron mass. The electron field is here described by a four-component function ψ. The so-called 'gamma matrices' γ^{μ} are four numerical matrices (4x4 arrays of given numbers) which have to be multiplied in a standard mathematical way with the components of ψ. (Actually, we have suppressed an extra component index on ψ to prevent the notation from becoming even more involved).

Analysis of the equation revealed the meaning of the four components of the Dirac field. It includes the description of the somewhat mysterious property called *spin*, best described as some intrinsic rotational degree of freedom. We could say that the electron is the quantum equivalent of a tiny spinning top - and it can be left- or right-handed. Remarkably, the equation turned out not just to describe the two spin components of an electron, but also the two spin states of another particle with exactly the same mass but the opposite (positive) charge. This particle is therefore called the positron. C.D.Anderson discovered this first example of an 'antiparticle' experimentally in 1932. It became clear that in fact all particles in nature have antiparticles, with exactly opposite properties such that when a particle and an antiparticle meet, the pair can annihilate each other and be converted into pure energy in the form of electromagnetic radiation - a dramatic instance of the equation $E=mc^2$. Because of the peculiar spin properties, the four-component object ψ is called a *spinor* rather than a vector. A further analysis of the Dirac equation also led to an explanation of Pauli's exclusion principle. This rule, obeyed by electrons and all other particles described by a Dirac-type equation, decreed that two or more of these particles could never sit in exactly the same state. This was a crucial but up to then ad hoc ingredient of quantum theory, needed to explain the periodic table of atoms. Indeed, as the electrons in an atom cannot all sit in the same lowest energy state, they have to systematically fill up the higher energy levels, causing different types of atoms to display entirely different chemical behavior.



QUOTE OF THE WEEK

This result is too beautiful to be false; it is more important to have beauty in one's equations than to have them fit experiment.

— *Paul Dirac*